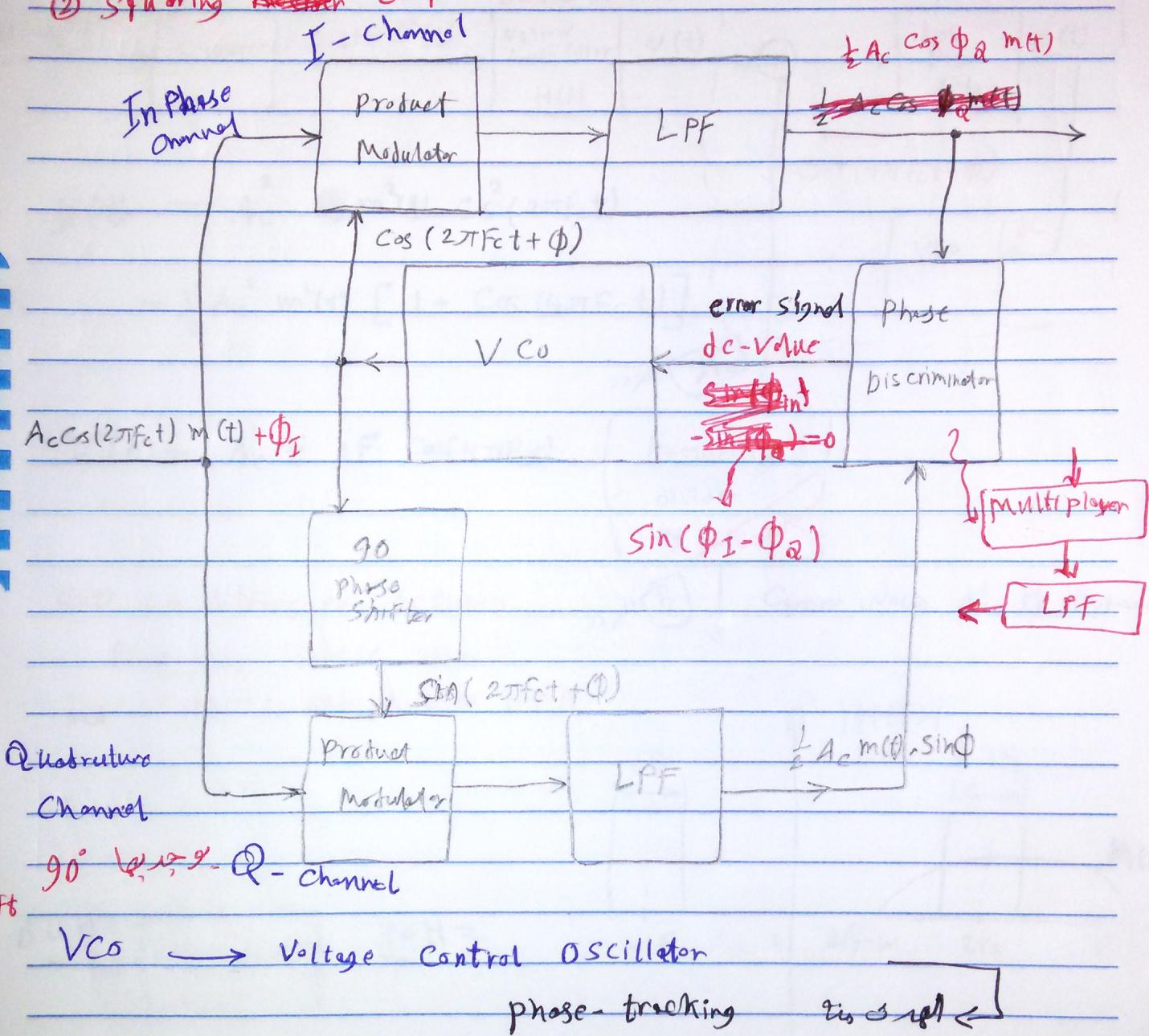


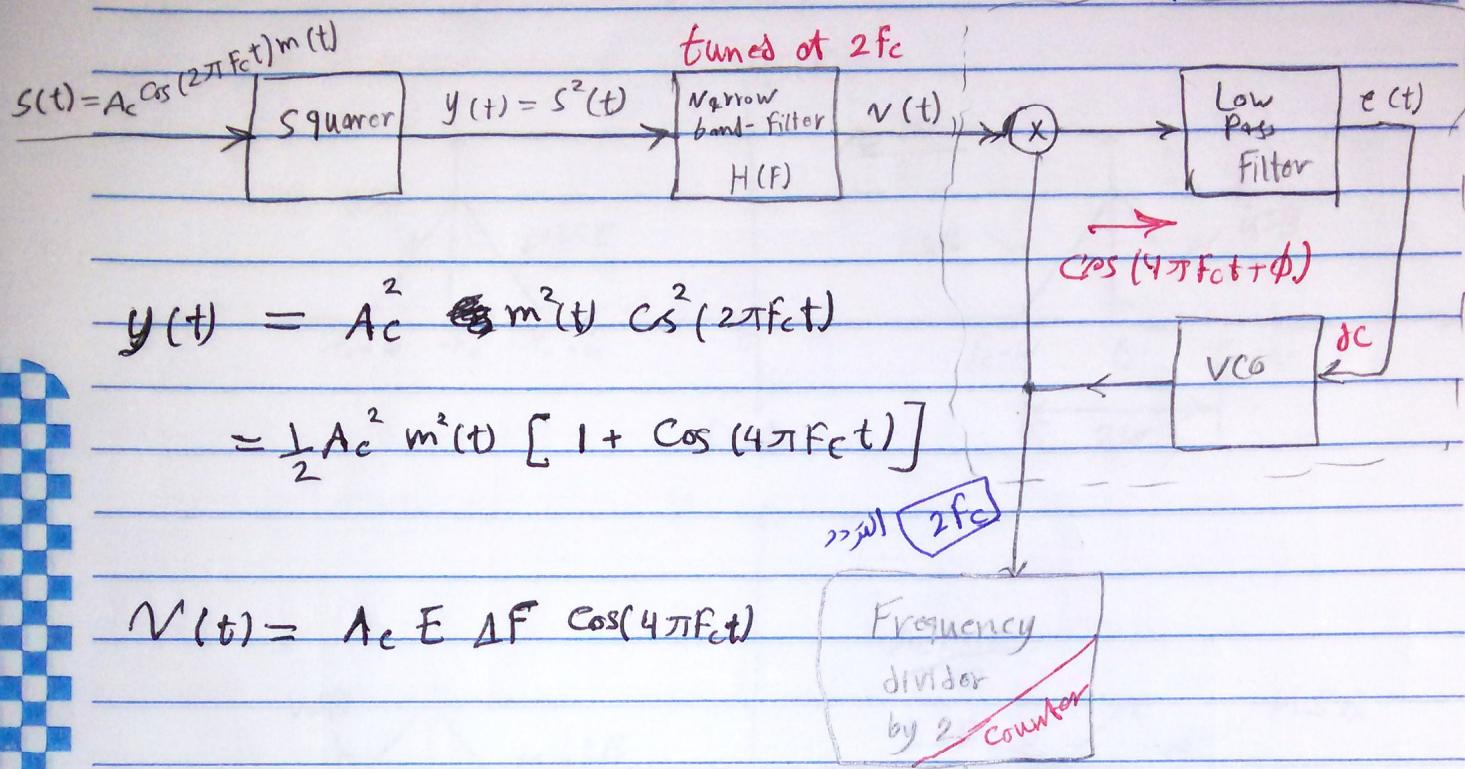
Using phase-shift lock

① Costas Receiver

② Synchronizing Loop



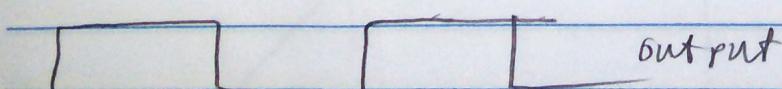
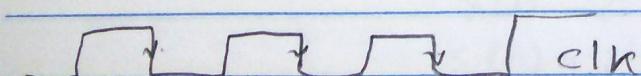
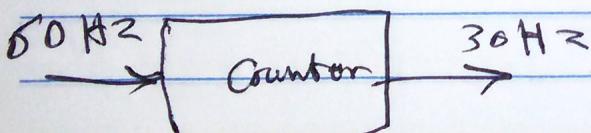
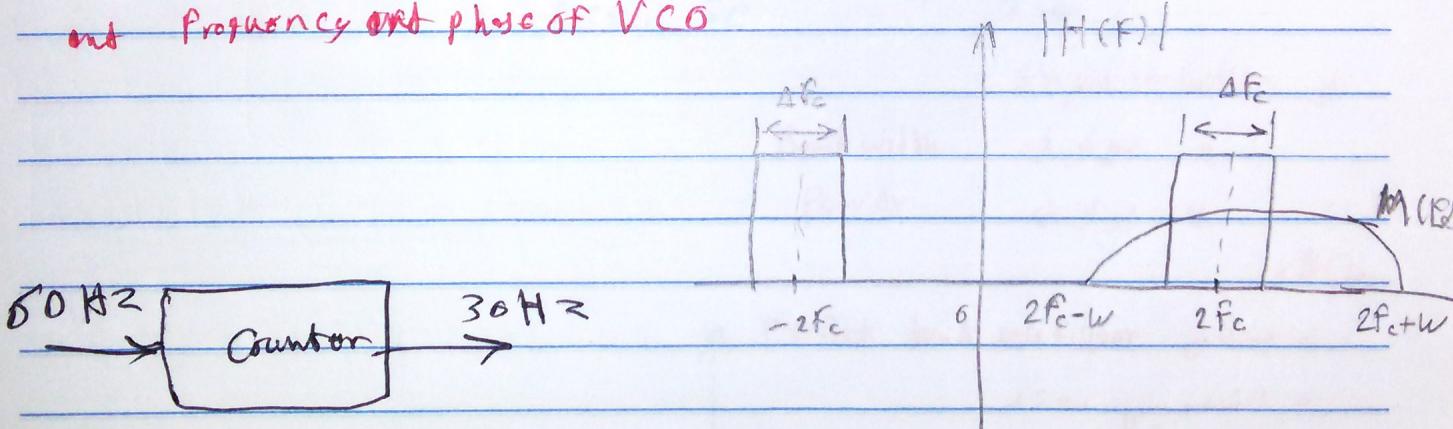
② Squaring Loop



$e(t) \rightarrow$ difference between
Frequency (phase $n(t)$)
and Frequency and phase of VCO

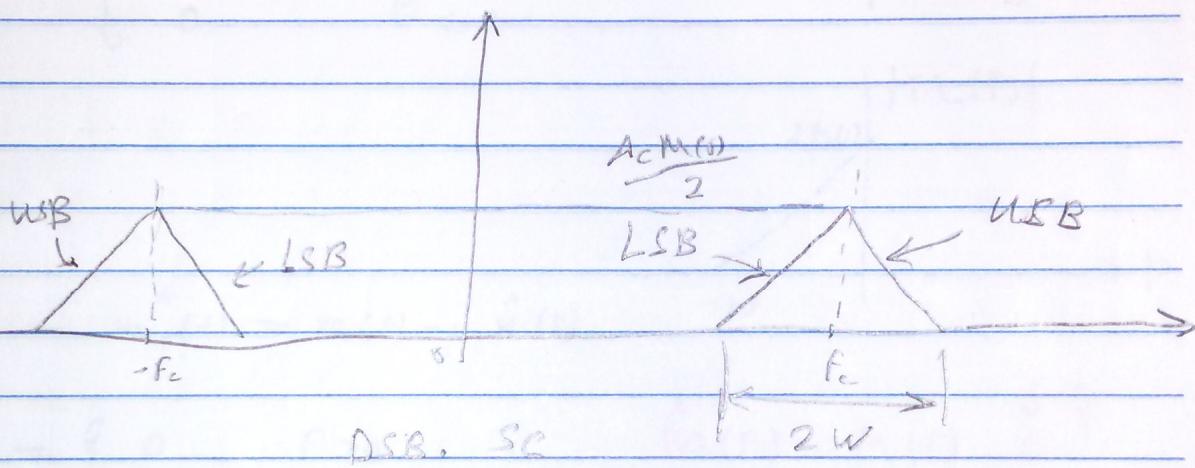
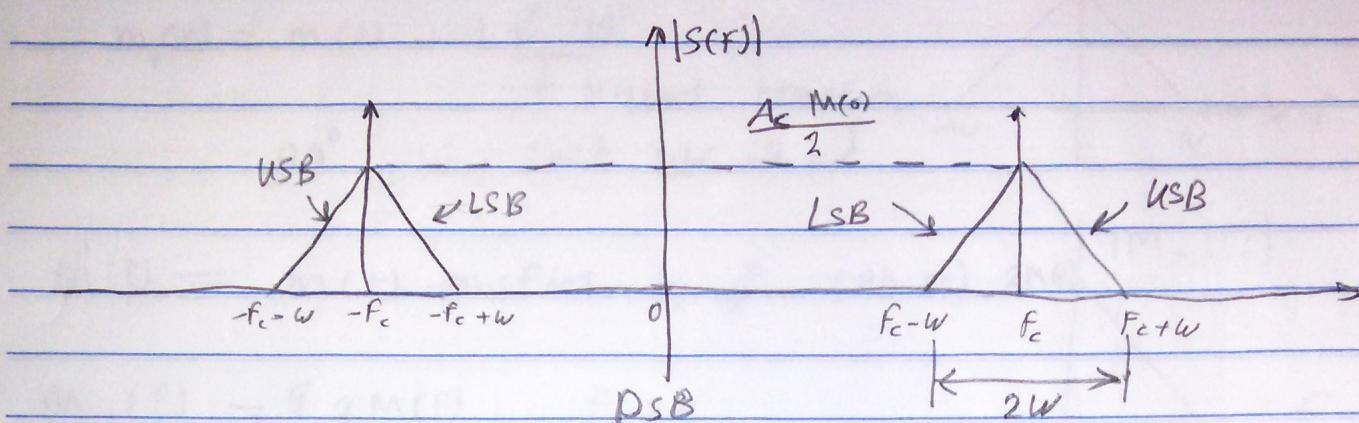
$\gg \omega$

Carrier wave at f_c frequency



6 bit $\otimes 0$

Single Side band Modulation (SSB)



Single Sideband \Rightarrow

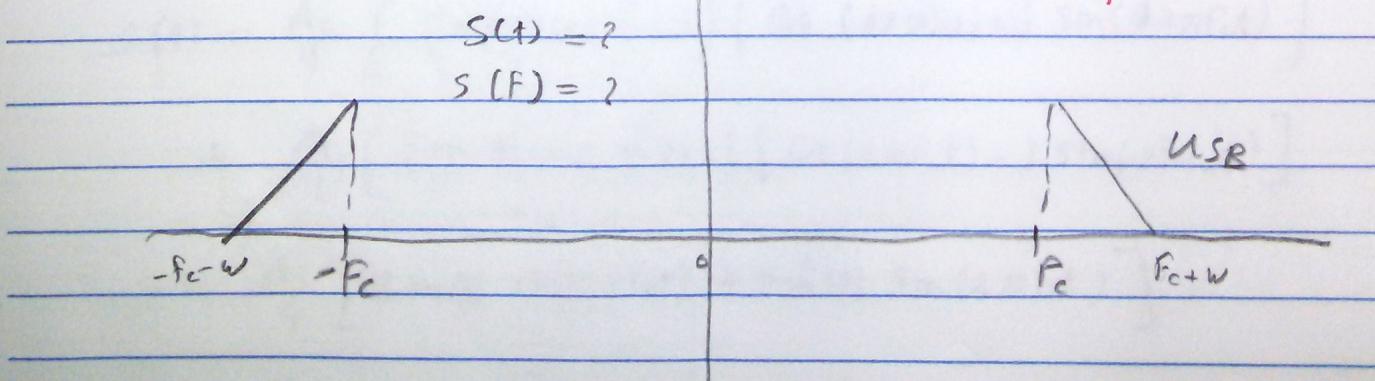
Band width \Leftrightarrow ω *

Power \Leftrightarrow P *

$\frac{P}{2} C_{DSB}$

Perfect band pass filter \Rightarrow ω *

less power *

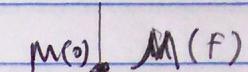


pre-envelope of $m(t)$:

$$m_+(t) = m(t) + j \hat{m}(t)$$

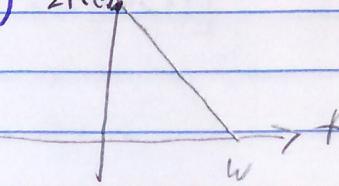
↓ Hilbert transform

90° phase shift $\text{left} \leftrightarrow \text{right}$



$$\hat{m}(t) = m(t) \text{ shifted by } \frac{\pi}{2} \text{ (Phase)} \quad |M_+(f)|$$

$$M_+(f) = \begin{cases} 2M(f) & f > 0 \\ 0 & f < 0 \end{cases}$$



~~Ans~~ =

$$m_-(t) = m_+^*(t) = m(t) - j \hat{m}(t)$$

$$\hat{M}(f) = M(f) \cdot e^{j \frac{\pi}{2}}$$

$$M_-(f) = \begin{cases} 0 & f > 0 \\ 2M(f) & f < 0 \end{cases}$$

To get single side band:

up or side band

$$s(t) = [m_+(t) e^{j2\pi f_c t} + m_-(t) e^{-j2\pi f_c t}] \frac{A_c}{4}$$

$$s(t) = \frac{A_c}{4} [(m(t) + j \hat{m}(t)) (\cos(2\pi f_c t) + j \sin(2\pi f_c t))]$$

$$+ \frac{A_c}{4} [(m(t) - j \hat{m}(t)) (\cos(2\pi f_c t) - j \sin(2\pi f_c t))]$$

$$= \frac{A_c}{4} [2m(t) \cos(2\pi f_c t) - 2\hat{m}(t) \sin(2\pi f_c t)]$$

$$= \frac{A_c}{2} [m(t) \cos(2\pi f_c t) - \hat{m}(t) \sin(2\pi f_c t)]$$

Lower Side Band

$$S(t) = \frac{A_c}{2} [m(t) \cos(2\pi f_c t) + \hat{m}(t) \sin(2\pi f_c t)]$$

For single side band

$$S(t) = \frac{A_c}{2} [m(t) \cos(2\pi f_c t) + \hat{m}(t) \sin(2\pi f_c t)]$$

Ex: $m(t) = A_m \cos(2\pi f_m t)$

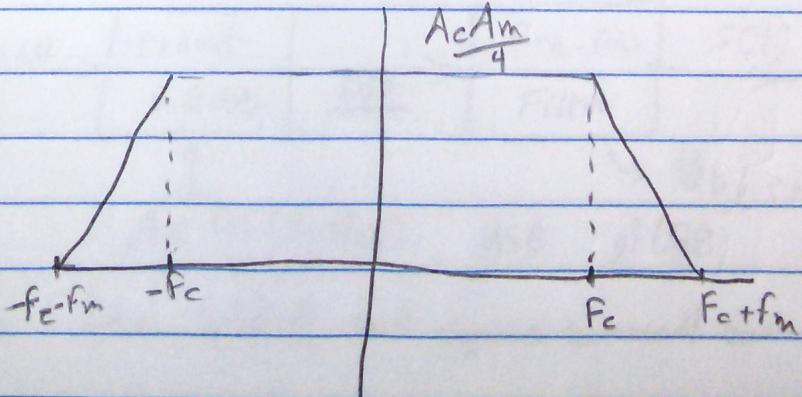
Ans USB Am ~~+~~

$$\hat{m}(t) = A_m \cos(2\pi f_m t + \frac{\pi}{2}) = A_m \sin(2\pi f_m t)$$

$$S(t) = \frac{A_c}{2} [A_m \cos(2\pi f_m t) \cos(2\pi f_c t) - A_m \sin(2\pi f_m t) \sin(2\pi f_c t)]$$

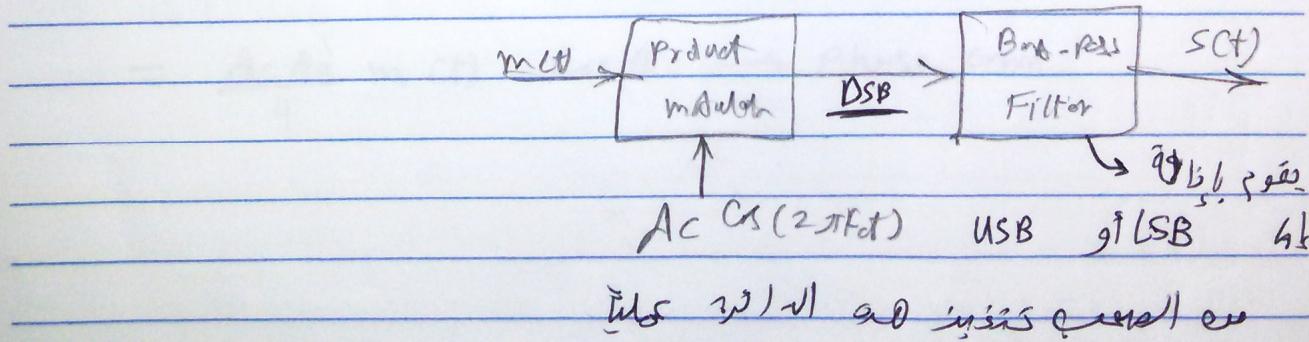
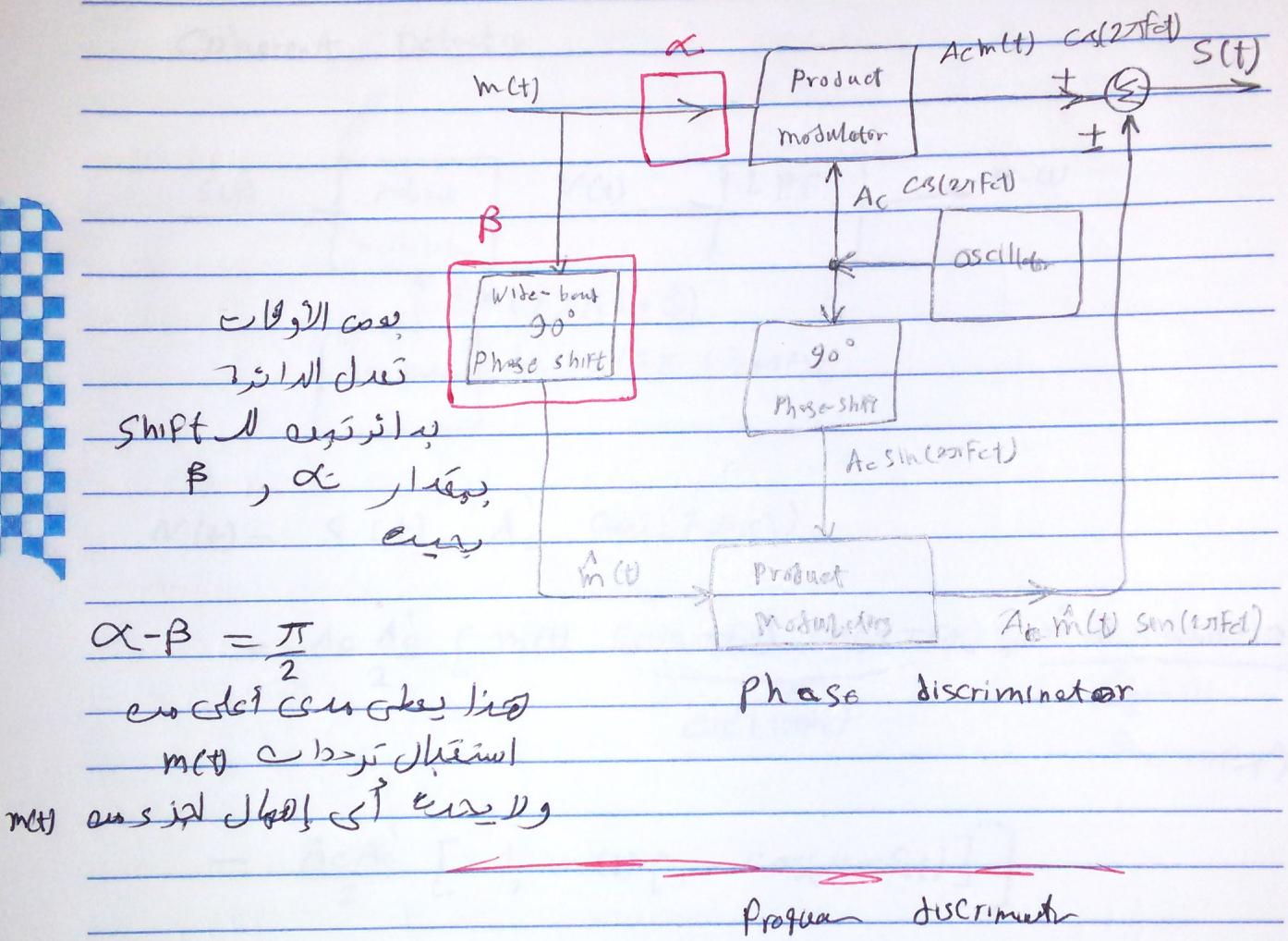
$$= \frac{A_c A_m}{2} \cos(2\pi (f_m + f_c) t)$$

$$S(f) = \frac{A_c A_m}{4} [\delta(f + f_c + f_m) + \delta(f - f_c - f_m)]$$



Generation of SSB Waves:

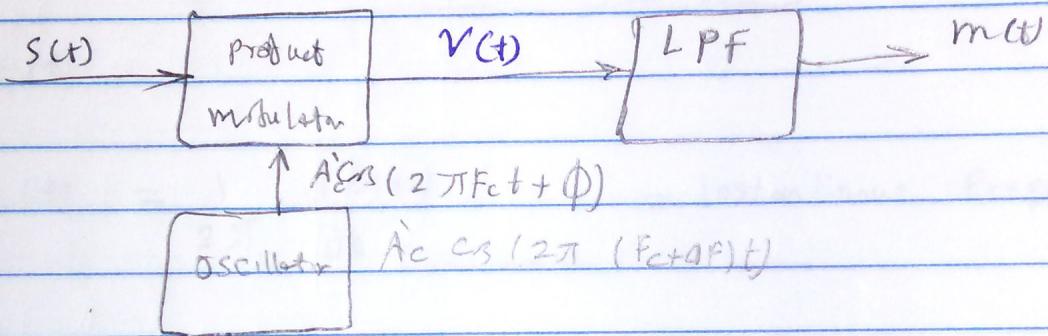
- ① phase discriminator
- ② Frequency discriminator



De modulation of SSB :

$$S(t) = \frac{A_c}{2} [m(t) \cdot \cos(2\pi f_c t) \mp \overset{1}{m}(t) \sin(2\pi f_c t)]$$

Coherent Detector



$$V(t) = S(t) \cdot A_c \cos(2\pi f_c t)$$

$$= \frac{A_c A_c'}{2} [m(t) \cos(2\pi f_c t) \cos(2\pi f_c t) \mp \overset{1}{m}(t) \sin(2\pi f_c t) \cos(2\pi f_c t)]$$

$$= \frac{A_c A_c'}{2} [\cos^2(2\pi f_c t) \overset{1}{m}(t) \sin(4\pi f_c t)]$$

$$= \frac{A_c A_c'}{2} [\frac{1}{2} m(t) [1 - \cos(4\pi f_c t)]]$$

$$= \frac{A_c A_c'}{4} m(t) \cos \phi \rightarrow \text{Phase error}$$

Angle Modulation

$$S(t) = A_c \cos(\theta_i(t))$$

Am wave

$$S(t) = A_c [1 + k_m m(t)] \cos(2\pi f_c t + \phi) \quad \boxed{= 0}$$

$$\theta_i(t)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \text{instantaneous frequency}$$

$$FM_{\text{wave}} \rightarrow f_c \quad PM_{\text{wave}} \rightarrow \phi$$

$$S(t) = A_c \cos(2\pi f_c t + \phi_c)$$

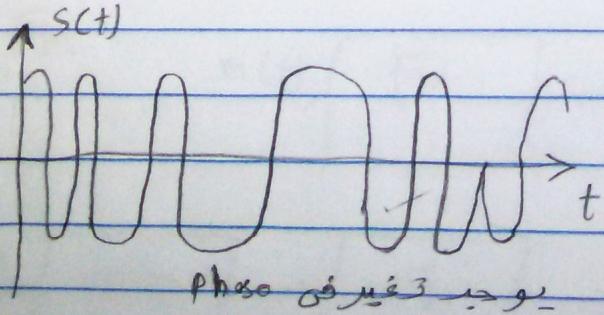
~~Frequency~~ \rightarrow ~~un modulated~~ \rightarrow ~~un modulated~~
~~Angle~~

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

$$PM_{\text{wave}} \rightarrow \theta_i(t) = 2\pi f_c t + k_p m(t)$$

$k_p \rightarrow$ phase sensitivity

$$S(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$



$$f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$$

$$\phi_i(t) = 2\pi \int_0^t f_i(t) dt$$

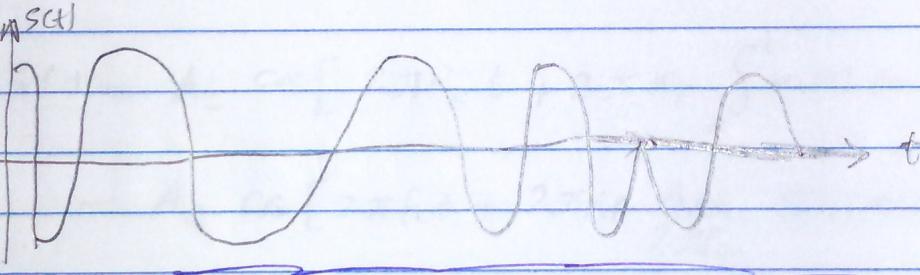
$$FM \rightarrow f_i(t) = f_c + k_p \cdot m(t)$$

k_p → frequency sensitivity

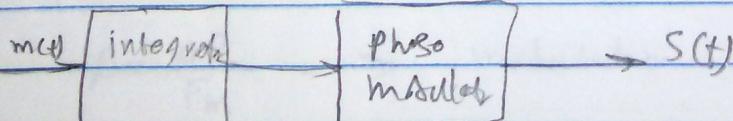
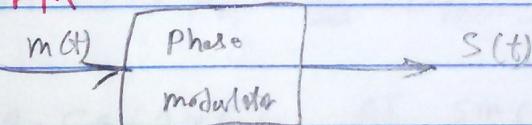
$$\phi_i(t) = 2\pi \int_0^t [f_c + k_p m(t)] dt$$

$$\phi_i(t) = 2\pi f_c t + 2\pi k_p \int_0^t m(t) dt$$

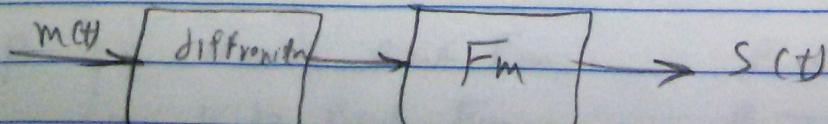
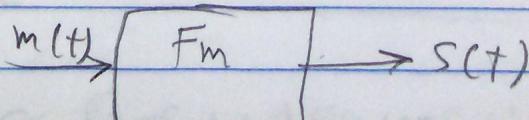
$$s(t) = A_c \cos [2\pi f_c t + 2\pi k_p \int_0^t m(t) dt]$$



FM from PM



PM Pm FM



FM and PM \rightarrow better than \rightarrow AM
 noise \rightarrow noise \rightarrow less noise
 less Amplitude \rightarrow less noise

With side bands, will cancel noise

FM Type:

- ① Single tone FM
- ② Multi tone FM

* Single tone:

$$m(t) = A_m \cos(2\pi f_m t)$$

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi K_p \int_0^t m(t) dt \right]$$

$$= A_c \cos \left[2\pi f_c t + 2\pi K_p \frac{A_m}{2\pi f_m} \sin(2\pi f_m t) \right]$$

$\Delta f = \pm K_p A_m$ ~~is~~ Frequency ~~deviation~~ deviation

$$s(t) = A_c \cos \left(2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \right)$$

$$\beta = \frac{\Delta f}{f_m} \rightarrow \text{modulation index}$$

$$\text{Frm} = f_c \pm K_p A_m = f_c \pm \Delta f$$

$$s(t) = A_c \cos \left[2\pi f_c t + \beta \sin(2\pi f_m t) \right]$$

$$\beta \rightarrow \begin{cases} \text{Narrow Band FM} & \beta < 1 \\ \text{Wide Band FM} & \beta > 1 \end{cases}$$

Narrow Band FM Wave

β is very small

$$S(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)]$$

$$- A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)]$$

$$\simeq \beta \sin(2\pi f_m t)$$

$$S(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t) \sin(2\pi f_m t)$$



$$S(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c [\cos(2\pi(f_c + f_m)t) - \cos(2\pi(f_c - f_m)t)]$$

$$B.W = 2f_m$$

* Wide Band frequency Modulation

$$\beta \gg 1$$

$$s(t) = A_c \cos [2\pi f_c t + \beta \sin (2\pi f_m t)]$$

$$= A_c \operatorname{Re} \{ e^{j2\pi f_c t + \beta \sin (2\pi f_m t)} \}$$

$$= A_c \operatorname{Re} \{ e^{j2\pi f_c t}, e^{j\beta \sin (2\pi f_m t)} \}$$

$$\hat{s}(t) = e^{j\beta \sin (2\pi f_m t)} \rightarrow \text{Complex Envelope}$$

$\hat{s}(t) \rightarrow$ periodic signal with period ~~$\frac{1}{2f_m}$~~ $\frac{1}{f_m}$

Using Complex F.S.

$$\hat{s}(t) = \sum C_n e^{j2\pi n f_m t}$$

$$C_n = \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \hat{s}(t) \cdot e^{-j2\pi n f_m t} dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j\beta \sin (2\pi f_m t)} \cdot e^{-j2\pi n f_m t} dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j[\beta \sin (2\pi f_m t) - 2\pi n f_m t]} dt$$

$$x = 2\pi f_m t$$

$$dx = 2\pi f_m dt$$

$$c_n = f_m \int_{-\pi}^{\pi} \frac{1}{2\pi f_m} e^{j[\beta \sin(x) - nx]} dx$$

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin(x) - nx]} dx \quad \text{Bessel Function}$$

$$c_n = J_n(\beta)$$

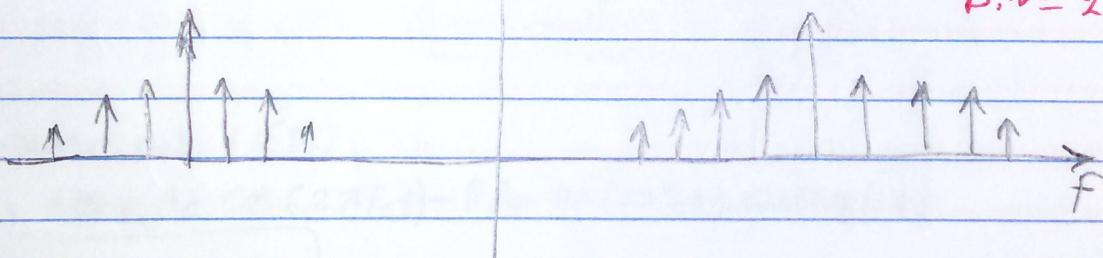
$$S(t) = \sum_{n=0}^{\infty} J_n(\beta) e^{j2\pi f_m t}$$

$$S(t) = A_c \operatorname{Re} \left\{ e^{j2\pi f_m t} \sum_{n=0}^{\infty} J_n(\beta) e^{j2\pi f_m t} \right\}$$

$$= A_c \sum_{n=0}^{\infty} J_n(\beta) \cos(2\pi(f_m + n f_m) t)$$

$|S(f)|$

$$B.W = 2\Delta f + 2f_m$$



$J_n(\beta)$ proportion

$$n = 0$$

$$J_0(\beta) = 1$$

$$n = \text{even}$$

$$J_n(\beta) = J_{-n}(\beta)$$

$$n = \text{odd}$$

$$J_n(\beta) = -J_{-n}(\beta)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

* Generation of FM Waves:

Transmission band width of FM Wave,

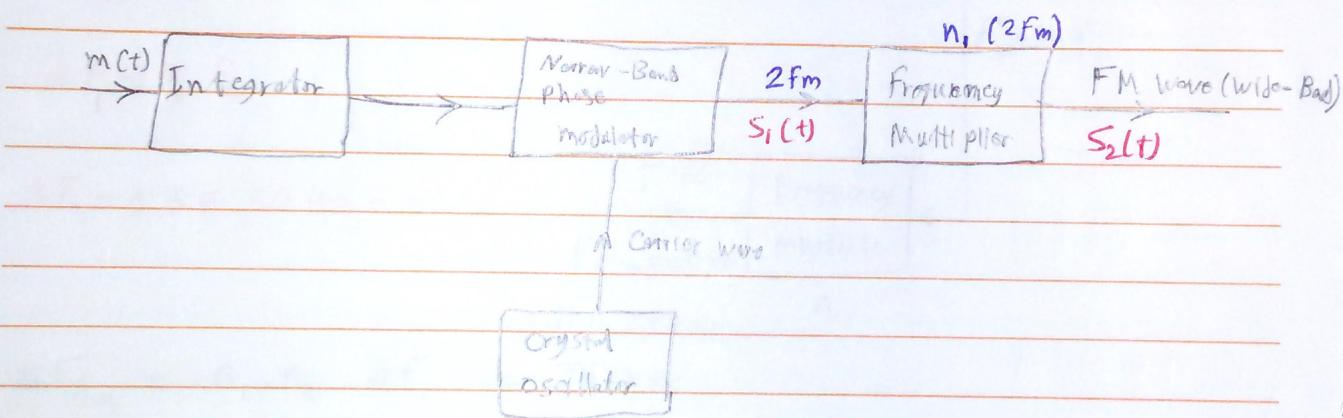
Corson's rule (Wide Band FM)

$$B.W. \approx 2\Delta f + 2f_m \approx 2n_{max}f_m$$

$$|J_n(B)| > 0.01$$

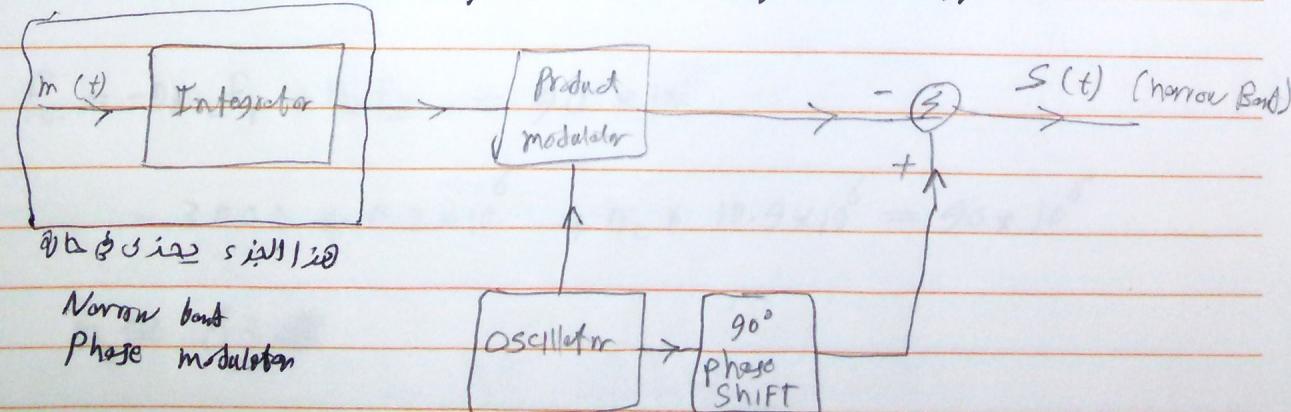
$$\Delta f = \frac{K_f A_m}{f_m} \approx \frac{n_{max} f_m}{f_m}$$

* Indirect ~~wide~~ FM generator:



Narrow Band (FM)

$$S(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

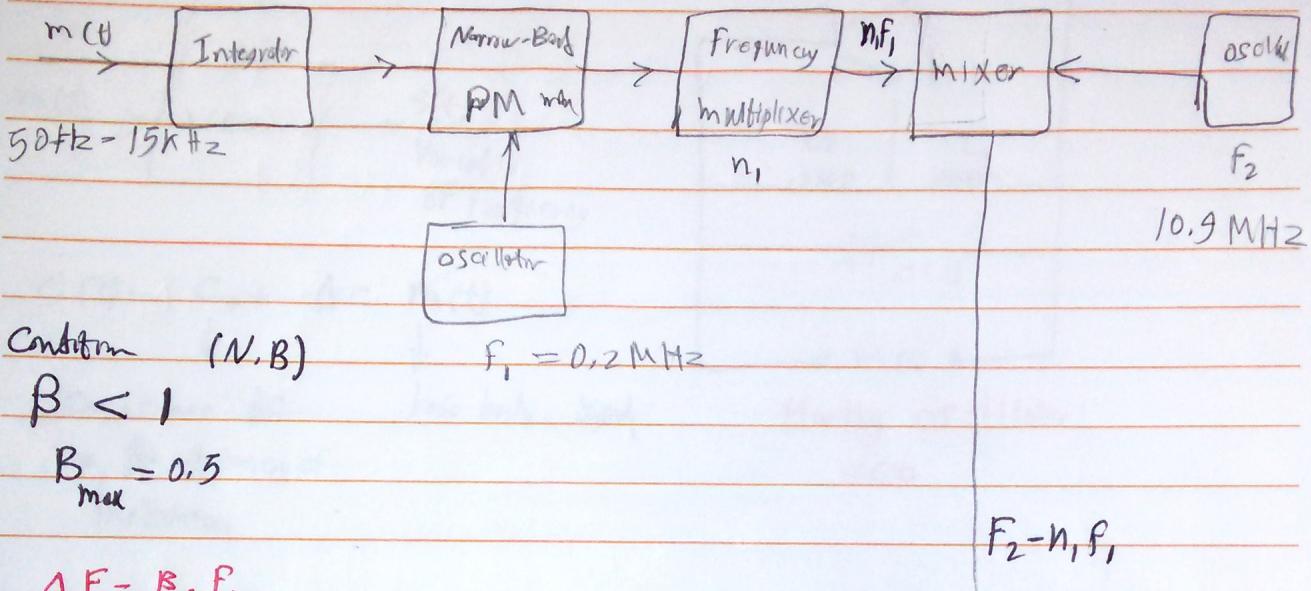


$$S_1(t) = A_1 \cos[2\pi f_c t + 2\pi K_f \int^t m(\tau) d\tau] \quad \text{if } m(t) = A_m \cos(2\pi f_m t)$$

$$S_1(t) = A_1 \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad \beta \ll 1$$

$$S_2(t) = A_1 \cos[2\pi f_c t + n_1 \beta \sin(2\pi f_m t)] \quad n_1 \beta \gg 1$$

Example : Find n_1, n_2



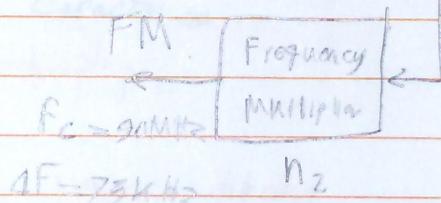
Condition (N.B)

$$\beta < 1$$

$$\beta_{\max} = 0.5$$

$$\Delta F_1 = \beta \cdot F_m$$

$$\Delta F_1 = 0.5 * 50 \text{ Hz} = 25 \text{ Hz}$$



$$\Delta F_{\text{prod}} = n_1 \cdot n_2 \cdot \Delta F_1 = 75 \text{ kHz}$$

$$\boxed{n_1 \cdot n_2 = 3000}$$

$$F_c = -n_1 n_2 F_1 + n_2 F_2 = 90 * 10^6$$

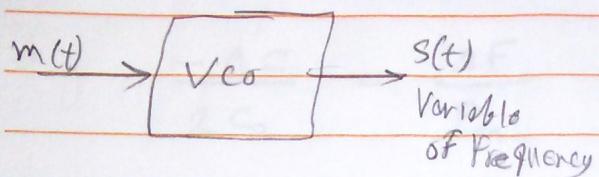
$$-3000 * 0.2 * 10^6 + n_2 * 10.9 * 10^6 = 90 * 10^6$$

$$n_2 \approx 63$$

$$n_1 \approx 48$$

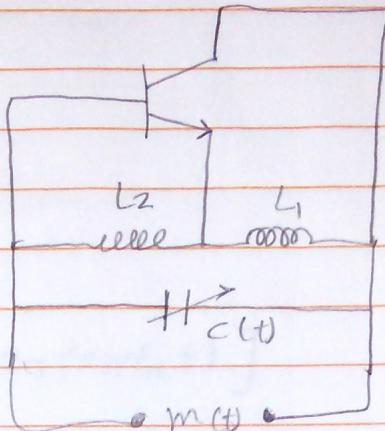
* Direct method, Direct FM generator

* Voltage Controlled oscillator



$$C(t) = C_0 + \Delta C m(t)$$

Capacitance in
the absence of
modulation



base band signal

Hartley oscillator

VCO

$\Delta C \rightarrow$ max. Change of Capacitance

It is constant in small pic

P.N Junction with reverse bias

~~PN~~ diodes \rightarrow Varactor, or Varicap

$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C(t)}} = \frac{1}{2\pi \sqrt{(L_1 + L_2) [C_0 + \Delta C m(t)]}}$$

$$1P m(t) = \cos(2\pi f_m t)$$

~~$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}}$$~~

$$f_i(t) = f_0 \left[1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) \right]^{-\frac{1}{2}}$$

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}}$$

$$F_i(t) = F_0 \left[1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) + \dots \right]$$

$$\boxed{\Delta C < C_0}$$

$$\frac{-\Delta C}{2C_0} = \frac{\Delta F}{f_m}$$

$$\therefore F_i(t) = F_0 \left[1 + \frac{\Delta F}{f_m} \cos(2\pi f_m t) \right]$$

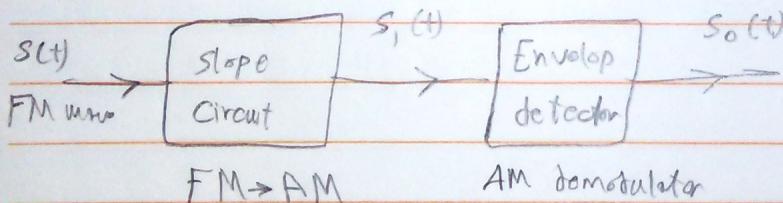
Instantaneous Frequency of FM wave

Disadvantage \rightarrow * Unstable Frequency
* Variable Capacitance is not Linear

* Detection of FM waves

- ① Slope detector
- ② PLL \rightarrow Phase Locked - Loop

① Slope detector



Slope Circuit \rightarrow Differentiator

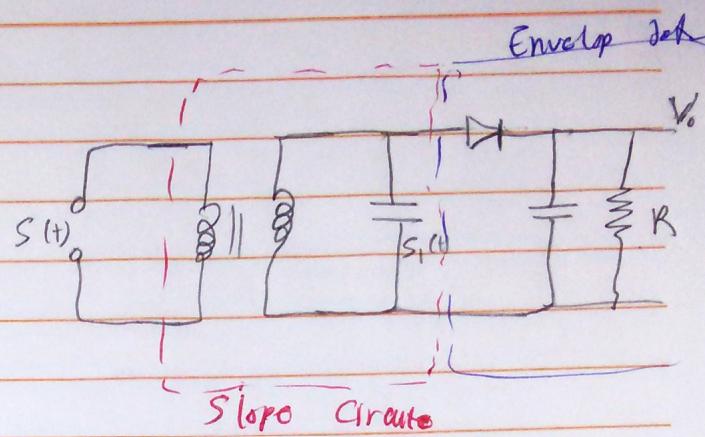
$$S_i(t) = A_c \cos [2\pi f_c t + 2\pi K_f \int_0^t m(t) dt]$$

$$S_i(t) = -A_c [2\pi f_c + 2\pi K_f m(t)] \sin [2\pi f_c t + 2\pi K_f \int_0^t m(t) dt]$$

$$S_o(t) = -A_c [2\pi f_c + 2\pi K_f m(t)]$$

$S_o(t) \propto m(t)$

Slope Circuite



② Phase - Locked - Loop :

